CIRCULATION MOTION OF A VISCOUS INCOMPRESSIBLE STREAM IN AN EXTRUDER CHANNEL

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An extruder in the form of a rectangular channel completely filled with an incompressible liquid is considered. The upper wall moves at an arbitrary angle to the channel axis. Equations for the circulation velocity and the flow lines in a channel cross section are obtained and studied numerically.

Let us represent the extruder in the form of a rectangular channel completely filled with a viscous liquid and having an upper wall moving uniformly with a velocity V_0 at an arbitrary angle β to its axis (Fig. 1). If the upper wall moves parallel to the channel axis one can calculate the longitudinal velocity field, as is done in [1, 3], for example.

We attempted to study the circulation flow and in particular to determine the velocity field and the flow lines of materials which from the point of view of rheology can be considered as a viscous incompressible liquid. We note that just this kind of motion plays a decisive role in the processing of plastics [4].

Since the conditions of shear do not depend on z, and we assume that the end effects are not significant (which can always be done when the channel is long enough), the velocity V will not depend on the longitudinal coordinate z. The Stokes equations and the continuity equation in the case of small Reynolds numbers have the form

$$\begin{aligned}\frac{\partial P}{\partial x} &= \mu\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}\right),\\ \frac{\partial P}{\partial y} &= \mu\left(\frac{\partial^2 V_y}{\partial x^2}+\frac{\partial^2 V_y}{\partial y^2}\right),\\ \frac{\partial P}{\partial z} &= \mu\left(\frac{\partial^2 V_z}{\partial x^2}+\frac{\partial^2 V_z}{\partial y^2}\right),\\ \frac{\partial V_x}{\partial x}&+\frac{\partial V_y}{\partial y}=0.\end{aligned} \tag{1}$$

Here μ is the dynamic viscosity.

We represent the pressure P in the form

$$P = p(z) + \Pi(x, y) \tag{2}$$

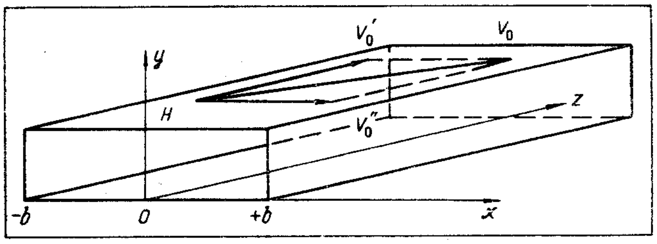

Fig. 1. Diagram of extruder with rectangular channel.

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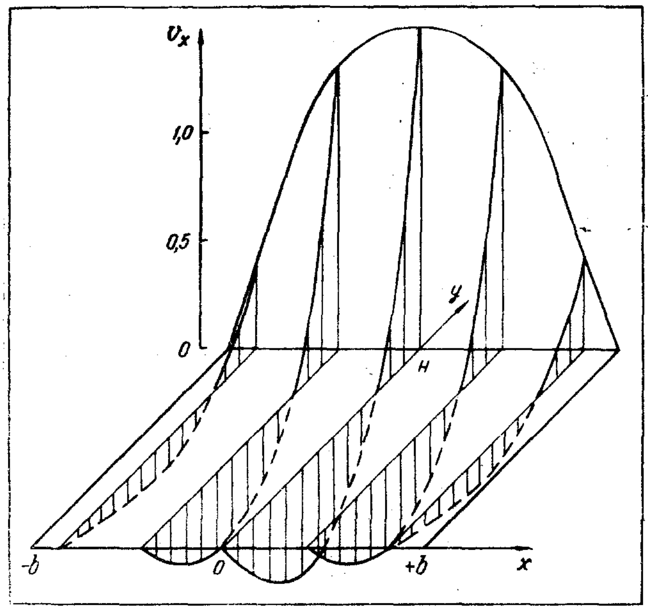

Fig. 2. Results of calculations of x component of the velocity v_x in some channel cross section z = const at H = 2b.

and then, substituting (2) into (1), we obtain the following system of equations:

$$\begin{cases} \dfrac{\partial \Pi}{\partial x} = \mu \Delta V_x, \\ \dfrac{\partial \Pi}{\partial y} = \mu \Delta V_y, \\ \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} = 0, \end{cases} \tag{3}$$

$$\begin{cases} \dfrac{dp}{dz} = \mu \Delta V_z, \\ \dfrac{dV_z}{dz} = 0 \end{cases} \tag{4}$$

with the boundary conditions

$$\begin{gathered} V_x(\pm b,\ y) = V_x(x,\ 0) = 0,\ V_x(x,\ H) = V_0'' = V_0 \sin\beta, \\ V_y(\pm b,\ y) = V_y(x,\ 0) = 0,\ V_y(x,\ H) = 0, \\ V_z(\pm b,\ y) = V_z(x,\ 0) = 0,\ V_z(x,\ H) = V_0' = V_0 \cos\beta. \end{gathered} \tag{5}$$

Thus, we see that the complete solution will consist of a linear superposition of the solutions of system (3) and system (4).

The solution of (4), using the boundary conditions (5), is presented in [1, 3].

The system (3) is easily converted into the form [2]

$$\Delta\Delta\psi = 0, \tag{6}$$

where

$$\frac{\partial \psi}{\partial x} = V_y, \quad \frac{\partial \psi}{\partial y} = -V_x. \tag{7}$$

We will seek a solution of (6) in the form of a series

$$\psi = \sum_n X_n(x) \cos\frac{\pi n y}{H} + \sum_m Y_m(y) \cos\frac{\pi m x}{2b}, \tag{8}$$

where the functions $X_n(x)$ and $Y_m(y)$ have the form [3]

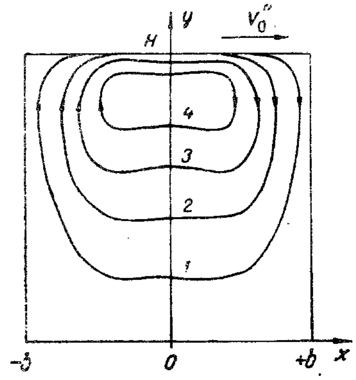

Fig. 3. Flow lines for different values of ψ.

$$X_n(x) = A_n \operatorname{ch} \frac{\pi n x}{H} + B_n \frac{\pi n x}{H} \operatorname{sh} \frac{\pi n x}{H} + C_n \operatorname{sh} \frac{\pi n x}{H} + D_n \frac{\pi n x}{H} \operatorname{ch} \frac{\pi n x}{H},$$

$$Y_m(y) = E_m \operatorname{ch} \frac{\pi m y}{2b} + F_m \frac{\pi m y}{2b} \operatorname{sh} \frac{\pi m y}{2b} + G_m \operatorname{sh} \frac{\pi m y}{2b} + J_m \frac{\pi m y}{2b} \operatorname{ch} \frac{\pi m y}{2b}. \qquad (9)$$

Using the boundary conditions (5) of system (3) and Eqs. (7), (8), and (9) we obtain an infinite system of linear algebraic equations relative to the coefficients A_n, ..., E_m, ...

Such a system can be solved only for finite values of m and n using electronic computers.

If we set m = n = 1 we obtain a system of eight equations which we will not present because of their awkwardness, but as an illustration we show in Fig. 2 the results of calculations of the x component of the velocity $v_x = V_x/V_0''$ in some channel cross section z = const at H = 2b.

It is seen from the graphs presented that the velocity v_x changes sign in practically all the cross sections x = const, which indicates the vortex nature of the motion.

Furthermore one can note that the maximum value of v_x is close to 1.5. This is explained by the fact that the values m = n = 1 were used in the calculations. With an increase in the number of terms of the series the value of v_x will approach 1, but in this case the volume of the calculations is considerably increased.

Using the results obtained it is easy to construct flow lines for whose family the equation is ψ = const.

The flow lines corresponding to the values $\psi = -0.05$ (curve 1), $\psi = -0.15$ (2), $\psi = -0.20$ (3), and $\psi = -0.25$ (4) are shown in Fig. 3.

In conclusion we note that the transverse velocity) as is seen from Eq. (6)) for small Reynolds numbers does not depend on the viscosity, and in addition the spatial trajectory of the particles represents a spiral whose pitch depends on the z component of the velocity.

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